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THE ORIGIN OF MATHEMATICS—A FIRST LESSON IN SECONDARY MATHEMATICS

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Introduction: The nineteenth century was characterized by an unparalleled activity in the realm of science. When Darwin and his followers succeeded in popularizing the doctrine of evolution, an amazing impetus was given to research in almost every field of human thought and endeavor. It soon became natural to go back to first principles, to study the growth and development of all existing things, to look for causes, motives, connections, controlling forces. New sciences sprang up over night. Innumerable questions were addressed to nature by carefully arranged experiments. The bulk of scientific knowledge increased immensely. And, as if by magic, even the past gave up many of its long concealed secrets. The geologic record of the rocks became legible. It revealed upheavals extending through aeons of time. The incredibly slow unfolding of life on this globe became established by countless fossilized remains. It was found that human civilization, too, had a long, prehistoric career. The story of man's painful upward struggle is still very fragmentary, but new data are coming to light every year. Many of our most precious human achievements are now known to have their roots in this early prehistoric period. It is becoming clear that many arts and sciences owe their origin to universal stimuli and to fundamental human needs. And when man finally succeeded in leaving written records on stone or papyrus, an almost modern state of mentality had been reached. The human mind, says a recognized scholar, has not changed essentially since palaeolithic time.

And now, Babylonia and Egypt, Europe and America, are furnishing something like a diary of this more modern period. Forgotten and dead languages are beginning to live again. The past is waking up. Our big museums, here and abroad, are flowing over with the treasures of buried ages.

What has all that to do with the subject of this article?

Merely this: Will the school find it profitable and feasible to be in harmony with this spirit of research and evolutionary insight? In particular, can a universally important subject like mathematics, the oldest of all the sciences, be made more interesting, more purposeful, more real and vital, by being *consciously* linked with the great causative agencies that called it into being and that led to its wonderful growth?

Fortunately, this question has been answered affirmatively by a host of educational leaders. Nor is it necessary to depend *merely* on the monumental contributions of Cantor or his predecessors. Many others have felt the inspiration of the evolutionary, historical point of view. In England, Branford did conspicuous educational work along this line. France has had men like Tannery; Germany had its Simon and Klein; Italy, its Boncompagni and Loria.

It is gratifying that America has had an honorable share in creating this new emphasis on historical mathematical studies. Thus, Hilprecht's volume on the Nippur tablets (1906) has been called "the greatest discovery of all time relating to Babylonian mathematics," comparable to the translation of the famous Ahmes papyrus of Egypt. *Rara Arithmetica* (1908), by Professor D. E. Smith, threw new light on familiar arithmetical processes. Smith and Karpinski's *Hindu-Arabic Numerals* (1911) is the most authoritative account written on the subject. That little gem, *Number Stories of Long Ago* (1919), also by Professor Smith, urgently calls for a sequel in geometry and algebra. Fine's *Number-System of Algebra* (1890) included an extensive historical sketch. Karpinski's translation of the algebra of Al-Khowarizmi (based on the Latin translation due to Robert of Chester) appeared in 1915. Stamper's *A History of the Teaching of Elementary Geometry* was published in 1909 (Columbia University). It offered a fine bibliography. The National Geometry Syllabus (1912) contained an history introduction prepared by Florian Cajori.

In fact, a very respectable literature on the history of mathematics and its pedagogic significance is now available. Even a brief review of the most important titles would fill many pages. As early as 1874, Hankel insisted on considering the development of mathematics as an integral part of the history of civilization. Gradually, school books began to include historical material. A pioneer effort was made in the excellent text of Baltzer, now out of print (sixth edition, 1883). Poul La Cour wrote a Danish text (1898) with a complete historical setting. In 1904, at a convention of Swiss secondary teachers, a resolution was passed demanding that greater attention be given to historical considerations. Paul Tannery's *Notions Historiques* appeared in 1903. During the same year, Tropfke's valuable topical history of elementary mathematics was completed. Branford's *A Study of Mathematical Education* was issued in 1908, being hailed by Nunn as "the most important and original of recent English contributions to the pedagogy of mathematics." Its point of view was entirely evolutionary. A few months later, Gebhardt's memorable "program" appeared, with its valuable summary and its excellent new suggestions. The International Commission on the Teaching of Mathematics (Rome, 1908) helped very distinctly to stimulate interest in historical studies. The reports of its numerous sub-committees have served, up to the present moment, to stress basic questions so often ignored by the schools.

More recently, the prehistoric and primitive phases of all the arts and sciences have received much attention. Those who are interested in "first beginnings" will find the new material accumulated by the scientific workers in ethnology, anthropology, and comparative sociology, both fascinating and illuminating. Such institutions as the American Museum of Natural History (New York), the Field Museum (Chicago), the Museum of the University of Pennsylvania, and the United States National Museum (Washington), have amassed treasures that we are only beginning to appreciate. (See, for example, *Source Book in Anthropology,* by A. L. Kroeber and T. T. Waterman, University of California Press, 1920.)

"That is all very well," the inexperienced teacher may say, "but before I can do anything in this direction, I wish to hear a more definite statement of the precise purpose of this historical material, and I must also have some idea as to the best method of using it."

The following paragraphs will attempt to answer these natural queries very briefly.

Let us consider, first, the cultural and practical aims of historical studies. They may be included under these headings:

1. *Comprehension.* Without the history of mathematics it is impossible to understand the relative significance and the organic interrelations of its concepts, principles, propositions, and applications.

2. *Appreciation.* Only through the history of mathematics can we fully realize the unique relation of mathematics to the origin and growth of pure and applied science and to the whole scheme of human existence. Said Napoleon to Laplace (1812): ''The welfare of nations is closely connected with progress in mathematics.'' (L'avancement et la perfection des mathématiques sont intimement liés a la prospérité de l'Etat.)

3. *Economy.* The history of mathematics enables us to avoid the errors of the past and to select methods which are economical and fruitful.

4. *Perspective.* Rich historical backgrounds keep us from falling an easy prey to the popular slogans of the hour. A person possessing historical perspective will avoid extremes. Progress is seen to be due to the constant interaction of theory and practice. Because Egypt and Rome had a passion for merely *practical* efficiency, and excluded speculation, they produced very few intellectual leaders and their civilization soon became stagnant. Purely ''theoretical'' studies very often yielded instruments of incomparable value (*e. g.,* conic sections—astronomy, engineering; fractional and negative exponents—logarithms; complex numbers—electrical theory).

5. *Patience.* The ''golden period of Greek mathematics'' knew nothing of ''minimum essentials'' and ''consumer's needs.'' The ''three great problems'' have a history of many centuries. Archimedes did not shout ''Eureka'' after a brief ''exposure'' to ''first year mathematics.'' The Greeks knew that there is no royal road to learning except real *work.* John Napier gave most of his life to the problem of simplifying arithmetical operations. Fermat's Theorem is not proved to this day. The temple of mathematics was reared by many generations. Those who would enter it must be prepared to tarry in its halls and to read its countless tablets with patience and singleness of purpose.

Finally, a few suggestions may be added as to methods. The writer has experimented in this field of instruction for years.

At the beginning of the course, development lessons and brief talks or illustrated lectures (lantern slides) are desirable. These must be followed later by occasional remarks and references. In the junior high school, the first ten lessons in geometry and algebra are largely historical in character. They are essentially of the form presented in the following pages. Pupils always enjoy this work. It is "different," and by creating a new interest it gives a good start to the school year. The introductory lesson, on the origin of mathematics, has been tried out by the writer in numerous classes (in Rochester, Cleveland, The Lincoln School of Teachers' College, Washington). It has also been frequently repeated by his colleagues. (The outline which is given here was used in the seventh grade. It is based on the writer's personal notes and on reports of observing teachers. Historical details were omitted in this printed summary, for obvious reasons.) All teachers who have experience in this work will render a distinct service by sending their comments or suggestions to the editor of this journal.

LESSON I. THE ORIGIN OF MATHEMATICS

Time, 40 Minutes

Purpose of Lesson. To develop the topic, *The Origin of Mathematics,* and to suggest the great importance of mathematics.

Plan. Age of Subject. Hence its necessary character. Situations causing quantitative questions. Their natural character. Necessity of counting. Situations demanding *measurement.* Natural origin of *arithmetic* and *geometry,* and hence of mathematics.

OUTLINE OF LESSON

(T = Teacher)
(P = Pupil)

Part I. Age of Mathematics. Its Necessary Character

T. What subject are we about to study?

P. Mathematics.

T. When you are asked to study any new subject, what questions concerning that subject naturally come to your mind?

P_1. We wish to know whether it is worth while.

P_2. We want to know "what it is all about."

P_3. We wonder how long it will take to master it.

P_4. We wish to know how it started.

T. Yes, every thoughtful pupil should ask these questions. I cannot answer all of them at once. But I hope that they will all be answered as you go on in your work. Today I shall take up but one of these questions, namely, how mathematics started. You will then begin to see why it is so important. What topic, therefore, are we to discuss today?

P_1. The beginning of mathematics.

P_2. The age of mathematics.

P_3. How mathematics originated.

T. In what books do we find information about the origin, or beginning, of our institutions, our government, our customs?

P. In historical books.

T. There are very interesting books on the history of mathematics. If you were able to read them, you would soon learn that mathematics is the oldest science invented by man. In fact, it is thousands of years old. It is so old that no one can tell just when it started. And the history books tell you that in every country we know about, even thousands of years ago, the people had some knowledge of mathematics. (A brief reference to Babylonia, to Egypt, and to primitive tribes, is desirable at this point.) If that were the only fact you knew about mathematics, what would it indicate to you?

P. It is a very necessary (important, essential) subject.

T. Yes, and we shall try to see what makes it so necessary. Are people in the habit of keeping a useless thing for thousands of years?

P. No, mathematics must have been very necessary at all times.

T. Now, if mathematics is such an old subject, and if we find it among all races at all times, what thought does that bring up?

P_1. We wonder how it started.

P_2. We wish to know what made it so necessary.

T. Exactly. Let us think about these questions: How mathematics began, and what made it so necessary. In the first place, do you suppose that some wise man invented it?

P. No, because so many people always used it.

PART II

NATURAL ORIGIN OF MATHEMATICS. SHORTAGE SITUATIONS. QUANTITATIVE QUESTIONS.

T. I shall try to make you see how *natural* it was for people to invent mathematics. It will not be hard to show you this. Last year I was in New York. One day we read that all freight trains had stopped running. When we heard that, what was our impression?

P. You thought of how you were going to get food.

T. Yes, and also coal, and many other things. People cannot let such a situation go on without trying to remedy it. So Mayor Hylan appointed a commission to look into the food situation. What did they do?

P. They found out about the food.

T. Yes, and first they went to the storehouses. What did they ask there?

P. "*How much* food have you on hand?"

T. What was the next thing to be found out?

P. *How many* people were in the city.

T. How did they find out?

P. They had to count them, or look up the census report.

T. How did the managers of storehouses find out the amount of food they had?

P. They counted the supplies on hand.

T. After they found out how much food they had on hand, and how many people there were, what were they able to tell?

P. *How long* the food would last.

T. What were the three questions that had to be answered?

P. *How much* food have we? *How many* people are in the city? *How long* will the food last?

T. How were the answers to these questions found?

P. By counting (by "figuring," etc.).

T. Now you will probably say that we do not *ordinarily* ask these questions. Why not?

P. We usually have "plenty of food."

T. Under what circumstances, therefore, should we *not* ask these questions?

P. When there is no shortage.

T. Why do we not ask, "How much air have we?"

P. We know we have "plenty."

T. But that is not always the case. Last year the newspapers reported a situation which *did* make it necessary to ask, "How much air have we?" Do you recall that situation?

P. A submarine became stuck in the mud under water.

T. Why did the sailors worry about the air in that case?

P. They could not live without air.

T. We certainly must have food and air. But there are many other situations similar to the ones we have talked about. When do we ask the questions you have mentioned?

P. When there is a shortage of something that we must have in order *to live*. (Refer to starving children of Europe at this point.)

T. What other shortage except a food shortage have you heard of?

P. A shortage of houses.

T. Yes, New York needs 100,000 new apartments, and many new houses are needed *here*. How did they find it out?

P. By counting ("estimating," etc.).

T. How can the house shortage be remedied?

P. By building new houses.

T. Is building as cheap as it used to be?

P. No, all the prices have gone up.

T. Can you give the reason for this?

P. Building material and labor cost so much more.

T. Yes. But why is building material, such as lumber, so much more expensive?

P. There is a shortage of lumber.

T. That is true. Let us consider that situation for a moment. Years ago this country was covered with forests. There was no shortage of lumber then. In fact, we are told that farmers used black walnut rails for fences. Today we know almost exactly how many walnut trees remain in any locality. Why didn't they count them formerly?

P. They had a great many trees.

T. Repeat the questions that arise naturally when there is a shortage.

P. *How much* have we? *How many* people must be taken care of? *How long* will the supplies last?

PART III

COUNTING AND MEASURING

T. Notice all these questions again. What causes people to ask these and similar questions?

P. Shortage situations.

T. Now you will readily understand that as soon as these questions:

How much?
How many?
How long?

were being asked, people had to think of some way of answering them. But before we go on, tell me whether we could possibly avoid asking these questions *today?*

P. (All agree that many everyday situations bring up these questions. Imagine a child going to school in the morning. "*How much* time have I to get to school? *How far* is it to the school? *How much* money must I take along?" Father and mother ask similar questions many times a day. "*How much* does sugar cost? *How many* pounds can I get for one dollar?")

T. (Again calls attention to necessary and natural character of these questions, and hence emphasizes the natural origin of mathematics.)

T. Of course, all these questions have to be answered. What is the only way of answering a question beginning with "*how many*"?

P. (One pupil, a girl, suggests "*so* many." Another child again refers to *counting.*)

T. Remember that people could not count at first the way *we* do. But you can all see that they *had* to count. There is no other way of finding out the quantity of supplies you have on hand. We shall learn, by and by, how people counted. Tell me now what, in your opinion, was the final outcome of all this counting.

P. (After some hesitation, arithmetic is mentioned.)

T. Yes, counting produced arithmetic. And now recall for a moment what you said about the shortage of houses, that is so distressing just now. Of course, you all know why we are living in houses, and why such a shortage is so serious. You may tell me about it.

P. We must have houses for protection against heat and cold and wild animals.

T. So we may say that houses, as well as food and water and air, are really necessary if we wish *to live*. (Attention is then called to the fundamental human needs, with special emphasis on food, clothing, and shelter.) Suppose now that you are going to build a shelter. Building material, as you know, is not always easy to obtain, and prices have been going up for a long time. What question, therefore, becomes very important?

P. How large is the house going to be?

T. In other words, what item stands out as a very big one?

P. The size of the house.

T. And how would the size of the house be determined?

P. By measurement. (Repeated questions are necessary to secure this answer.)

T. At all times there were many occasions for measurement, just as there were many occasions for counting. You will understand, after all this discussion, that both counting and measuring are necessary and natural processes. They are the foundation of all mathematics; and if someone should ask you, now, how mathematics began, what would you say?

P. Mathematics began as soon as people had to count and to measure.

T. Let us consider one more question. What was the final outcome, in your opinion, of all the measuring that was necessary in so many situations?

P. (No pupil can suggest the correct answer. Some say, "arithmetic"; others, "mathematics." Finally, one boy mentions, "geometry.")

T. (Writes "geometry" on the blackboard, and analyzes its constituent parts, comparing the word with "geography." Thus, we have

geo—graphy and
geo—metry.

It is then seen that geometry really means "*earth measurement*" or surveying.)

PART IV

REVIEW AND SUMMARY

1. Outline: Origin of Mathematics. (Oral composition.)
    1. Age and extent.
    2. Importance.
    3. Shortage situations.
    4. Questions. (Food questions; shelter questions.)
    5. Answers.
    6. Counting—Arithmetic.
    7. Measurement—Geometry.
    8. The Foundation of Mathematics.

2. Key Words of Lesson:

   Age of Mathematics.
   Necessary—shortage situations.
   Natural—food, shelter, clothing.
   Counting—arithmetic.
   Measuring—geometry.

3. Written Summary:

The Origin of Mathematics

Mathematics is the oldest of all the sciences. This shows that it must have been very necessary. It began whenever people asked such questions as, how much? how many? how long? These question were asked whenever there was a shortage or a scarcity of things which we must have. The only way in which these questions could be answered was by counting and measuring. Counting led to arithmetic, and measuring led to geometry. Arithmetic and geometry are the foundation of mathematics.

4. Assignment: Pupils are to write a brief account of (1) the origin of mathematics, (2) the value of mathematics as demonstrated by the situation which would arise if all mathematics were suddenly destroyed. They are to find in news papers reports of shortage situations and bring these reports to school.